Properties of the Shapiro steps in the ac driven Frenkel-Kontorova model with deformable substrate potential

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Properties of the dynamical-mode-locking phenomena are studied in the ac driven overdamped Frenkel-Kontorova model with deformable substrate potential. Appearance of very large subharmonic steps due to deformation of the substrate potential significantly influences the stability and existence of harmonic steps. Strong correlation among harmonic and subharmonic steps has been observed in which the larger the width of half-integer steps, the smaller that of harmonic steps. Amplitude dependence of harmonic steps significantly changes with the deformation of the potential where deviation from the well-known Bessel-like oscillations appears. Strong influence of the frequency of the ac driving force on the appearance and size of subharmonic steps has been observed.

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I. INTRODUCTION

Since the first observation of dynamical-mode-locking phenomena, stability and properties of resonant solutions or Shapiro steps have been matter of many theoretical and experimental studies in systems such as charge-density wave conductors [1–3] and systems of Josephson-junction arrays biased by external currents [4–8]. In order to gain an insight into physics of these complex macroscopic many-body systems with competing interactions, the attention has been always focused on simple many-body models. Among these models, the dissipative (overdamped) Frenkel-Kontorova (FK) model is one of the simplest but still complex enough that can capture the essence of many physical phenomena.

The standard Frenkel-Kontorova (FK) model represents a chain of harmonically interacting particles subjected to a sinusoidal substrate potential [9]. It describes various commensurate and incommensurate structures that when subjected under an external driving force show rich dynamical behavior. In the presence of an external dc+ac driving force, the dynamics is characterized by the appearance of the staircase macroscopic response or the Shapiro steps in the response function $\bar{v}(\bar{F})$ of the system [10–12]. These steps are due to the dynamical mode-locking of the internal frequency that comes from the motion of particles over the periodic substrate potential with the frequency of an external ac force. Although the standard FK model has been very saucerful in the explanations of many phenomena related to the Shapiro steps such as amplitude or frequency dependence and the noise effects [13–16], it could not be used for the studies of any phenomena related to the behavior of subharmonic steps. It is well known that in the standard FK model, for commensurate structures with integer values of winding number only harmonic steps exist [17]. In the commensurate structures with noninteger values of winding number, besides harmonic, subharmonic steps appear, however their size is so small that they are invisible on the regular plot of the response function what makes analysis of their properties very difficult [10,11].

Contrary to the standard FK model, the large subharmonic steps can appear in the presence of the deformable substrate potential [18]. In the real physical systems, the shape of the substrate potential can deviate from the standard (sinusoidal) one, and this may affect strongly the transport properties of the system. In the physical situations, such as charge-density waves, Josephson junctions, or crystals with dislocations, application of standard FK model could be very restricted, and it is hard to believe that real physical systems could be "exactly" described by standard models or by employing perturbation methods. Introducing a family of nonlinear periodic deformable potentials, Remoissent and Peyrard [19] obtained in a control manner by an adequate choice of parameters rich variety of deformable potentials related to the physical systems such as Josephson junctions, charge-density wave condensates, and crystals with dislocations (these deformable potentials allow the modeling of many specific physical situations without employing perturbation methods). They have shown that the shape of the substrate potential was of great importance for the modeling of discrete systems [19].

In the present paper we will examine how the deformation of the potential influences the dynamical-mode-locking phenomena in the (dc+ac)-driven overdamped FK model particularly focusing on the amplitude dependence of the Shapiro steps. In order to select a type of deformable potential for our studies, our focus was on these deformable potentials that satisfy following requirements: it is necessary that the new form of potential refers to the same physical systems (charge-density wave systems and the system of Josephson junction arrays) as the overdamped FK model, that it is tunable, and that in the case of zero deformations, it reduces to the standard (sinusoidal) form that has been studied in our previous works [13–16].

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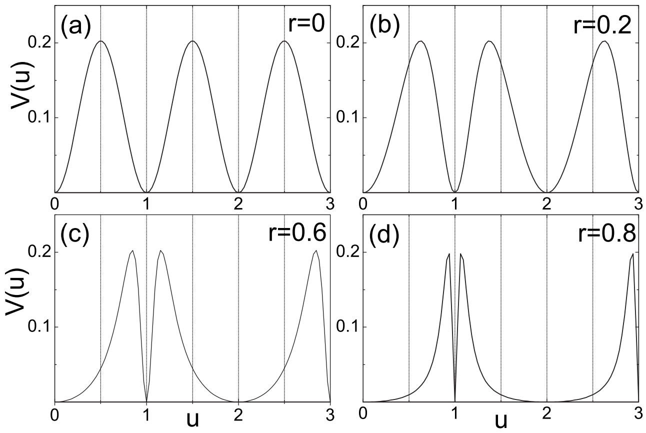

FIG. 1. Asymmetric deformable potential for $K=4$ and different values of the shape parameters r.

The obtained results have shown that the deformation of the potential strongly influences the stability and properties of the Shapiro steps. Strong correlation between the size of harmonic steps and the appearance of large half-integer steps has been observed. Appearance of subharmonic steps with the deformation of the potential changes the amplitude dependence of harmonic steps. Although the step width still has oscillatory dependence of amplitude, the form of oscillations strongly deviate from the well-known Bessel-like form. Strong influence of the ac frequency on the appearance of subharmonic steps and the amplitude dependence has been observed.

The paper is organized as follows. The model is introduced in Sec. II. Simulation results are presented and analyzed in Sec. III. Finally, Sec. IV concludes the paper.

II. MODEL

We consider the dissipative (overdamped) dynamics of series of coupled harmonics oscillators u_l driven by dc and ac forces:

$$F(t) = \bar{F} + F_{ac}\cos(2\pi\nu_0 t), \tag{1}$$

where $\bar{F}$ is the dc force while F_{ac} and $2\pi\nu_0$ are the amplitude and frequency of the ac force, respectively. The equation of motion is

$$\dot{u}_l = u_{l+1} + u_{l-1} - 2u_l - V'(u_l) + F(t), \tag{2}$$

where $l=-\frac{N}{2},\ldots,\frac{N}{2}$.

In order to include deformation of the potential, in the Eq. (2), we will replace sinusoidal potential $V(u_l)$ that has been studied previously [10–16] with the one from the family of parametrized deformable periodic potentials, the asymmetric deformable potential (ASDP) [19]:

$$V(u) = \frac{K}{(2\pi)^2}\frac{(1-r^2)^2[1-\cos(2\pi u)]}{[1+r^2+2r\cos(\pi u)]^2}, \tag{3}$$

where K is the pinning strength and r is the shape parameter $(-1<r<1)$. In Fig. 1, the ASDP is presented for different values of the shape parameter r.

This potential refers to the same physical systems as the overdamped FK model [19], and by an appropriate choice of the shape parameter, it can be tuned in a controlled manner from the simply sinusoidal (standard) potential for $r=0$ to an asymmetric periodic one for $0<|r|<1$ with a constant barrier height and two inequivalent successive wells with a flat and sharp bottom, respectively. The position u_b of the potential barrier is determined by the relation $\cos(\pi u_b)=2r/(1+r^2)$. Precisely, here the asymmetry means that the pining in the two successive potential minima is different. This type of potential is considered as a natural way to describe lattice with diatomic basis or dual lattices by generalizing the standard model that assumes simple sinusoidal potential [19]. In this model, particles during their motion interpolate between two media with different physical properties. The model has two energetically equivalent ground states, but these two states are not physically equivalent, in particular, they do not have the same dynamical properties [19]. The pinning of the particles strongly depends of the shape of potential well, and as it was shown previously [19], in the potential with sharp maxima and wide minima, even the very large kinks can be pinned.

When the system is driven by homogenous periodic force, the competition between two frequency scales (the frequency ν_0 of the external periodic force and the characteristic frequency of the motion over the periodic substrate potential driven by the average force $\bar{F}$) can result in the appearance of the synchronization phenomena (resonance). The ac force induces additional polarization energy into the system that is different from zero (less than zero) only when the velocity reaches the resonant values [10]:

$$\bar{v} = \frac{i\omega + j}{m}\nu_0, \tag{4}$$

where i,j and m are integers ($m=1$ for harmonic and $m>1$ for subharmonic steps). In the same time, the average pinning force will also be different from zero, and the system will get locked since the average pinning energy of the locked state (on the step) is lower than of the unlocked state. As $\bar{F}$ increases, the particles will stay locked until the pinning force can cancel the increase in $\bar{F}$.

Equations (2) have been integrated using the fourth-order Runge-Kutta method with the periodic boundary conditions for commensurate structure with the interparticle average distance (winding number) $\omega=\frac{1}{2}$ (ω is rational for the commensurate and irrational for the incommensurate structures). The time step used in the simulations was 0.02τ for lower values of r and 0.0002τ for $r>0.8$ (τ was the period of ac force). The force was varied with the step 10^{-4} (10^{-5} or 10^{-6} is used for the studies of very small subharmonic steps) and a time interval of 100τ was used as a relaxation time to allow the system to reach the steady state. The response function $\bar{v}(\bar{F})$, in particular the appearance and amplitude dependence of the steps are analyzed for the different frequencies and shapes of the substrate potential.

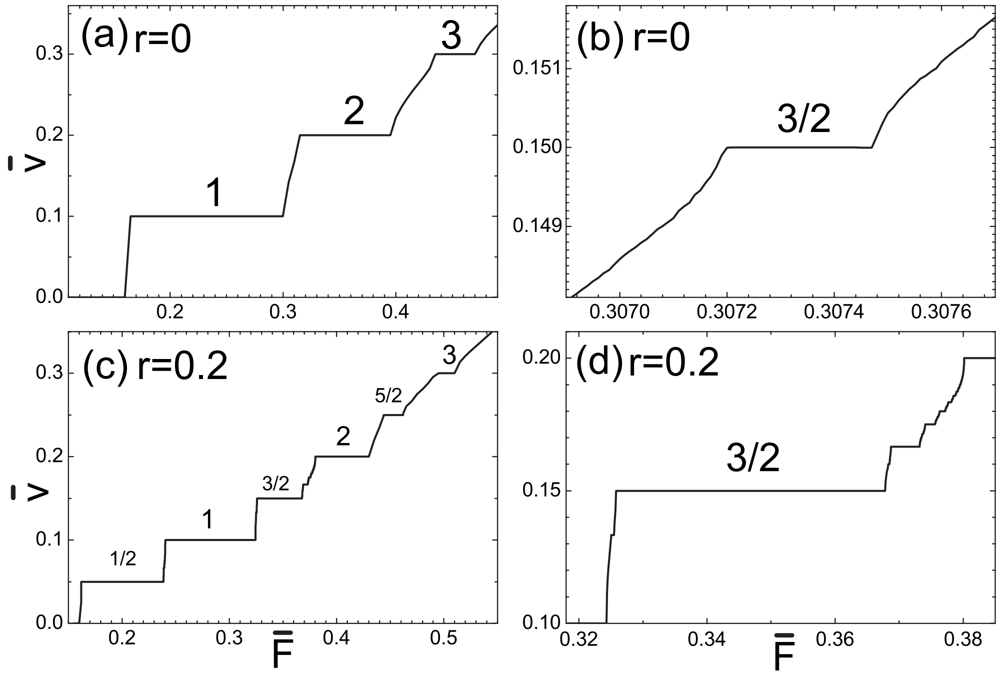

FIG. 2. Average velocity as a function of the average driving force for $\omega=\frac{1}{2}$, $K=4$, $F_{ac}=0.2$, $\nu_0=0.2$, and different values of the shape parameter $r=0$ and 0.2.

III. RESULTS

When potential gets deformed, very large subharmonic steps appear on the response function $\bar{v}(\bar{F})$. In Fig. 2, the response function $\bar{v}(\bar{F})$ for the commensurate structure $\omega=\frac{1}{2}$ is presented for two different values of the shape parameter r.

When $r=0$, for the standard case with sinusoidal potential in Fig. 2(a), the response function is characterized by the appearance of large harmonic steps. Beside the harmonic steps, in the commensurate structure $\omega=\frac{1}{2}$ subharmonic mode-locking also appears, however, the size of steps is so small that it is invisible in the Fig. 2(a). In Fig. 2(b), enlarged curve from Fig. 2(a) is presented, where the subharmonic step $\bar{v}=\frac{3}{2}\frac{\omega}{\nu_0}$ (the step width $\Delta F=0.000\,72$) can be seen. Contrary to the standard case, with the deformation of the potential very large half-integer and higher-order subharmonic steps appear as we can see in Fig. 2(c). In Fig. 2(d), the same step $\bar{v}=\frac{3}{2}\frac{\omega}{\nu_0}$ as in Fig. 2(b) is shown for $r=0.2$. The difference in size is obvious, for $r=0.2$, the step width ($\Delta F=0.042$) is significantly increased comparing with the case for $r=0$.

The appearance of subharmonic steps and the changes of critical depinning force due to deformation of the substrate potential in deferent commensurate structures have been studied in our previous work [18]. Origins of subharmonic steps have been matter of many debates and are still not well understood. What is shown by other studies [20–22] and also our work indicates is that presence of many degrees of freedom in the system plays an important role in the appearance of subharmonic mode-locking (the studies show that half-integer and higher subharmonic steps have different origins). It is well known that, standard FK model with winding number $\omega=1$ reduces to a single coordinate model, and as it was proven, cannot exhibit subharmonic mode locking [10,17,18]. However, if potential gets deformed, system cannot be described by single coordinate or single particle model since we have two groups of particles with different dynamical properties. As it was shown, deformation causes the appearance of whole series of large subharmonic steps, where in the limit of very large shape parameter (when $r\to 1$), dynamical mode locking disappears while critical depinning force F_c diverges (for some value of system parameters, F_c may even decrease at small values of r) [18]. In this work, we will particularly focus on the stability and amplitude dependence of the steps. Although amplitude and frequency dependence have been studied in detail in the standard FK model in our previous works [13–16], all of these studies have been dedicated only to the properties of harmonic steps. Contrary to these previous studies, working in a nonstandard FK model gives us a possibility to study not only the interference phenomena in more realistic situations but also to study the properties of subharmonic steps.

In Fig. 3, the width of the first harmonic ($\bar{v}=\frac{1}{1}\omega\nu_0$) and half-integer ($\bar{v}=\frac{1}{2}\omega\nu_0$) steps as a function of the shape parameter is presented.

As shape parameter increases, the size of the first harmonic step decreases reaching its minimum at $r=0.25$, after which it increases again, and then after reaching maximum at around $r=0.65$ decreases to zero as $r\to 1$. Meanwhile, the width of the half-integer step increases reaching its maximum at the same value of shape parameter $r=0.25$ that corresponds to the minimum of the harmonic step width (at this point the harmonic and fractional step are almost of the same size). These results show strong correlation between the steps in which the larger is the size of half-integer step the smaller that of harmonic step. We can also see in Fig. 3 that

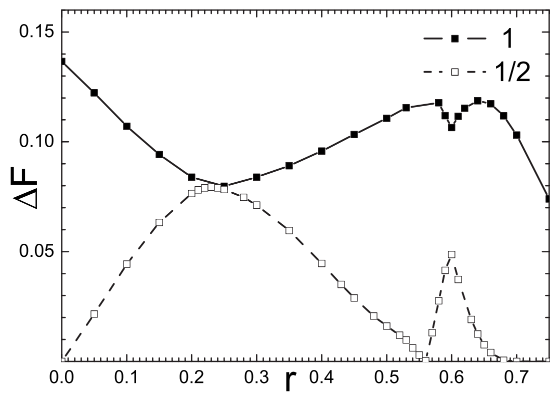

FIG. 3. The width ΔF of the first harmonic and fractional steps $\bar{v}=\omega\nu_0$, and $\frac{1}{2}\omega\nu_0$, respectively, as a function of the shape parameter r for $\omega=\frac{1}{2}$, $K=4$, $F_{ac}=0.2$, and $\nu_0=0.2$.

there is a second much smaller maximum of the half-integer step width and the corresponding minimum of the harmonic step width at $r=0.6$. The appearance of two maxima at the curve for half integer, and the corresponding minima at the curve for harmonic steps could be explained by the following process: for the commensurate structure $\omega=\frac{1}{2}$, we have two particles per one potential minima when $r=0$. As r increases and potential is more and more deformed as can be seen in Fig. 1, for values around $r=0.25$ in Fig. 3 that correspond to the first pair of corresponding minimum and maximum, the half of particles are still in sharp minima and the other half in the wide minima. With the further increase in r, as the sharp minima are getting more and more narrow in Fig. 1, for $r=0.6$ that corresponds to the second pair of corresponding minimum and maximum in Fig. 3, instead of two particles in every minima, there will be one particle in sharp and tree in wide minima. Finally, for very large deformations ($r\rightarrow 1$), the sharp minima practically disappear and all particles are strongly pinned in wide minima, what results in the disappearance of dynamical mode locking.

In Fig. 4, the amplitude dependence of the critical depinning force and the first harmonic step width for different values of the shape parameter is presented.

Changes of the amplitude dependence as r increases are directly related to the appearance of large half-integer steps. The appearance of large half-integer steps can be seen in Fig. 5, where their width as a function of amplitude for the same values of the shape parameter as in Fig. 4 is shown.

As we can see the appearance of half-integer steps is correlated with minima and not with absolute value of critical depinning force (presented by the dashed line).

Strong correlation between the harmonic and half-integer steps can be also seen in Fig. 6 where amplitude dependence of the first harmonic ($\bar{v}=\omega\nu_0$) and half-integer step ($\bar{v}=\frac{1}{2}\omega\nu_0$) for $r=0.2$ is shown.

The width of the half-integer step exhibits oscillations, where the maxima correspond to the minima of the curve for harmonic step. On the other side, the maxima of harmonic step correspond to the zero minima of fractional step since the harmonic steps always reach maximum values at the points where subharmonic mode locking disappears, and there are only harmonic steps in the response function $\bar{v}(\bar{F})$.

These results in Figs. 4 and 5 clearly show the impact that deformation of the potential has on dynamical-mode-locking phenomena. In the standard case for $r=0$, in the absence of half-integer steps, the amplitude dependence in Fig. 4 is characterized by the well-known Bessel-like form of oscillations. However, as r starts to increase and small half-integer steps appear for $r=0.05$ in Fig. 5, this behavior will change. The size of harmonic steps decreases while the minima become rounded and never go completely to zero meanwhile, the shape of oscillations still has the Bessel-like form. At $r=0.1$, as half-integer steps increase in Fig. 5, new maxima start to appear and the Bessel form of oscillations starts to change in Fig. 4. At $r=0.2$, for large half-integer steps, the width of harmonic step and the critical depinning force oscillate, however contrary to the standard case for $r=0$, maxima of one curve corresponds to maxima of another, while the form of oscillations became anomalous where the second maxima is lower than the third one. With further increase in r, at $r=0.5$ and 0.6, maxima of the oscillations are significantly reduced while periodicity is changing.

The physical process that stays behind Bessel-like oscillations of the step size with amplitude is the backward and forward displacement of particles induced by the ac force. Namely, in (dc+ac)-driven systems, dynamics is characterized by combination of two types of motions: linear motion in the direction of the dc force and the backward and forward jumps due to the ac force. Therefore, the particles perform motion that consists of series of backward and forward jumps, where the ac amplitude determines how much this motion is retarded [2]. In Fig. 7, the motion of one particle during one period of the ac force is presented.

If we consider a particle at the site i, then during one period, particle will first jump n sites backward, reach the $i-n$ site, and then hop again $n+1$ sites forward to the site $i+1$. During next period, it will repeat again these back and forward jumps and move to the site $i+2$. In that way, by repeating these back and forward jumps with every period of the ac force it will move. The distance (the number of sites n) over which particles moves is determined by the amplitude of the ac force [2]. For the values of the ac amplitude that correspond to the first maximum (in Fig. 4 for $r=0$), particles will spend most of the time on the site and then hop to the next well, while for the values at the second maximum, particles will jump one site back and two forward. As the ac amplitude increases, particles will hop between wells that are more and more distant while spending less time on the sites, and consequently, the step width will decrease.

Deformation of the potential will affect this backward and forward motion and by that the amplitude dependence of the step width and critical depinning force. Due to asymmetry of the potential, there are now two groups of particles and two different types of wells in which they can move. For the value of F_{ac} that corresponds to the first maximum, particles just jump to the next site where those from the wide jump to the sharp minima and vice versa. For the value of F_{ac} that

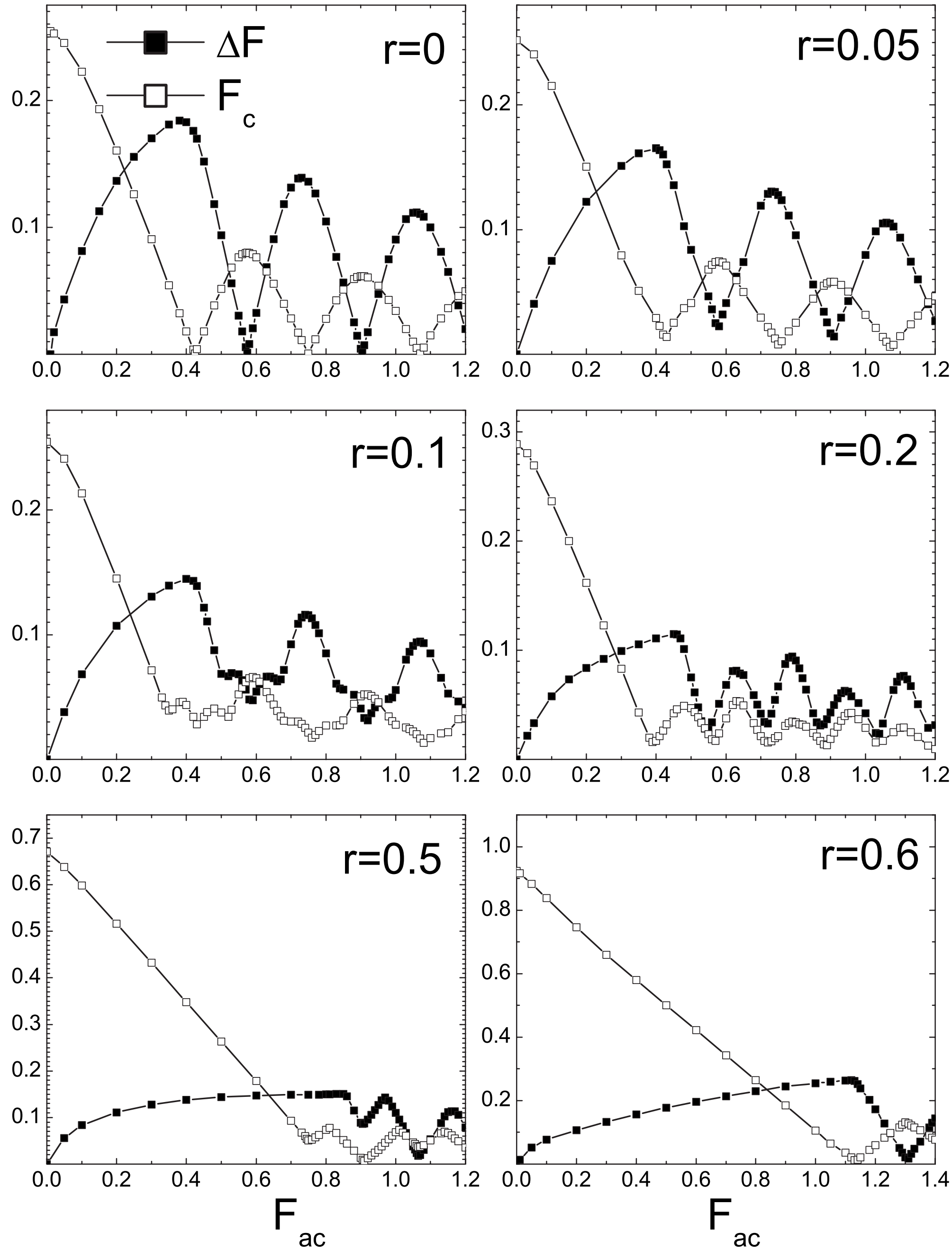

FIG. 4. The width ΔF of the first harmonic step and the critical depinning force F_c as a function of the ac amplitude for $\omega=\frac{1}{2}$, $K=4$, $\nu_0=0.2$, and $r=0$, 0.05, 0.1, 0.2, 0.5, and 0.6.

corresponds to the second and all even maxima, particles jump backward to the different type of minima what means that particles from sharp minima will jump back to the wide minima and from there forward to the wide minima at the site $i+1$. However, for the value of F_{ac} that correspond to the third and other odd maxima in Fig. 4, particle at site i will move backward to the same type of minima and from there forward to the different type of minima at the site $i+1$ what means particles from sharp minima will jump backward to the sharp minima again and from there forward to wide minima. Due to different pinning, the type of minima (sharp or wide) between which particle jumps will affect its motion and therefore the step size and the maxima of the oscillations.

Another effect that will also affect this backforward motion and therefore amplitude dependence is the changing of the number of particles in potential wells. As we already mentioned, for the commensurate structure $\omega=\frac{1}{2}$, we have two particles per one potential minima, that are all equivalent when $r=0$ for sinusoidal potential in Fig. 1(a). As r starts to increase, and the sharp minima are getting more and more narrow in Figs. 1(b)–1(d), the number of particles will change from two particles per minima for small values of r over one particle in sharp and three in wide minima to

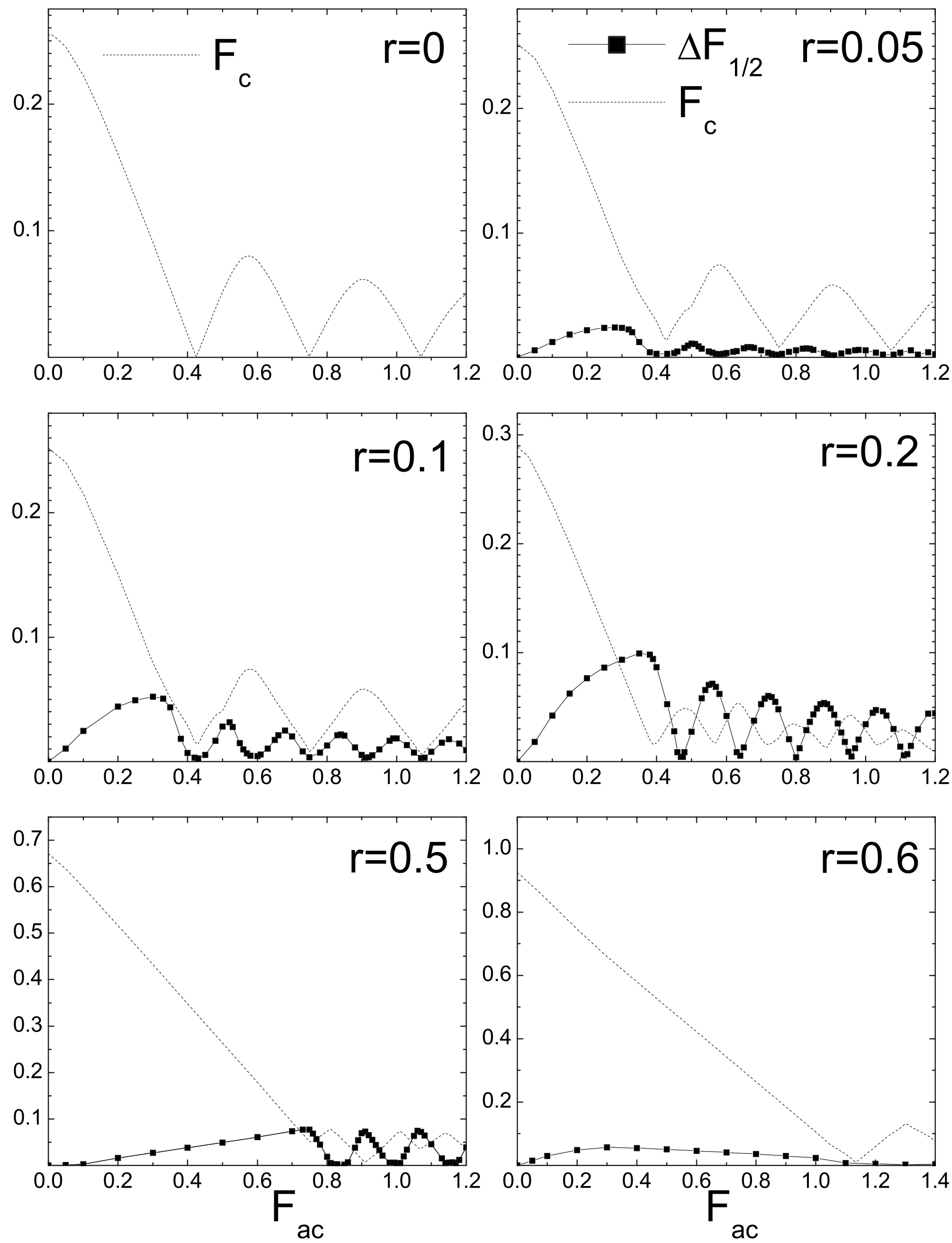

FIG. 5. The width ΔF of the half-integer step as function of the ac amplitude for $\omega=\frac{1}{2}$, $K=4$, $\nu_0=0.2$, and $r=0$, 0.05, 0.1, 0.2, 0.5, and 0.6. Dashed line represents critical depinning force F_c.

strongly pinned four particles in wide minima when $r\to 1$.

Besides the amplitude dependence, in Figs. 4 and 5, we can also see that the dynamical dc threshold (critical depinning force for dc driven system, in the absence of ac force when $F_{ac}=0$) increases with the deformation of the potential. In Fig. 8, the increase in dynamical dc threshold F_{c0} with the deformation of the potential is presented.

In the limit $r\to 1$, dynamical dc threshold diverges.

During our simulations we have noticed that subharmonic steps are much bigger if the frequency of applied ac force increases. In Fig. 9, subharmonic Shapiro steps for two different value of frequencies are presented.

We can clearly see the significant increase in the step number and size with the increase in frequency. This result is particularly important since frequency dependence of the Shapiro steps has been matter of many controversies. According to single coordinate models, step should be frequency independent at the high frequencies, meanwhile in systems with many degrees of freedom they show strong dependence of frequency where even oscillatory behavior appears in the high amplitude limit [13–16]. Our previous works in the standard FK model have shown that amplitude and frequency (period) of the ac force play analog role in the dynamical-mode-locking phenomena what raises the ques-

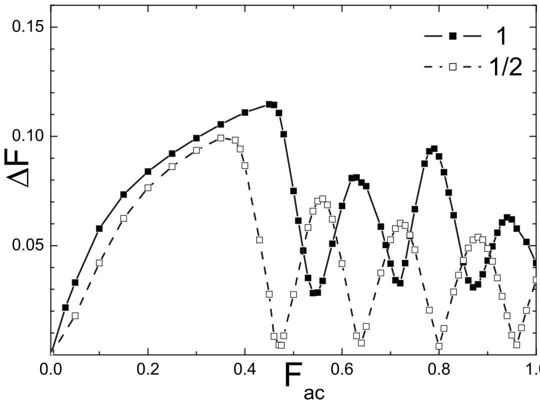

FIG. 6. The step widths ΔF of the first harmonic and the half-integer steps as a function of the ac amplitude F_{ac} for $\omega=\frac{1}{2}$, $K=4$, $\nu_0=0.2$, and $r=0.2$.

tion whether it is particularity of the standard FK model and harmonic Shapiro steps or it would appear also in models where subharmonic steps are present. Detail studies of the frequency dependence of subharmonic steps will be published separately.

The amplitude dependence of the harmonic, half-integer, and subharmonic steps obtained for $\nu_0=0.5$ is shown in Fig. 10 ($\bar{v}=\frac{i}{m}\omega\nu_0$, where $i=1$ and $m=1$ for harmonic, $m=2$ for half-integer, and $m=3$, 4, and 5 for subharmonic steps).

Contrary to the case at lower frequency in Fig. 4, at the higher frequency in Fig. 10(a), the size of maxima is increased where the shape of oscillations for the half-integer step better agrees with the Bessel function. These results, as the results in Figs. 4–6, show the strong correlation among the steps. We have been studied also the other commensurate structures ($\omega=1$), and we have always observed the same behavior.

Amplitude dependence and deviation from the Bessel-like behavior have been subject of many theoretical and experimental studies in charge density wave systems [23,24] and systems of Josephson junction arrays [25–27]. It was already suspected that a different choice of potential might distort somewhat the amplitude dependence of the Shapiro steps [24]. Fractional and integer Shapiro steps were found to be

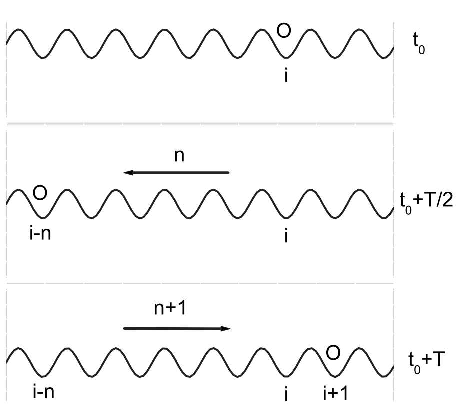

FIG. 7. The motion of a particle in sinusoidal substrate potential during one period of the ac force, where $n=0,1,2,\ldots$ is the number of sites over which the particle moves.

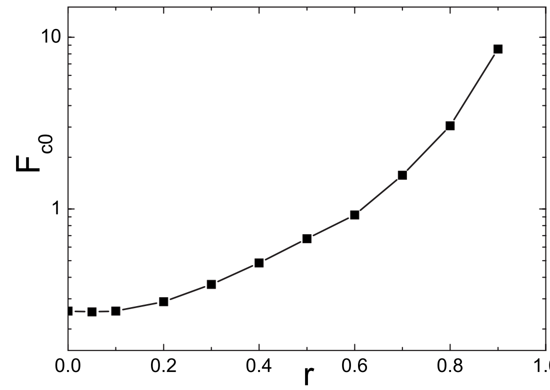

FIG. 8. Dynamical dc threshold F_{c0} as a function of the shape parameter r for $\omega=\frac{1}{2}$ and $K=4$.

correlated in which, the larger width of fractional steps, the smaller that of integer steps [25,27]. Harmonic and half-integer steps have been measured as a function of microwave amplitude and magnetic field in the high-T_c grain-boundary junctions, where, as in our case, appearance of half-integer steps at the minima of critical current (F_c in our model) has been observed and the amplitude dependence of steps has been classified into three different types (the behavior for the smallest, intermediate, and large half-integer step widths) [25]. In our examination of amplitude dependence, we have classified the same types of behavior, where this classification and comparison with these experiments [25] have been presented in our previous work [28], and therefore, they will not be repeated in this work. Anomalous amplitude dependence as in Figs. 4–6 for $r=0.2$, has been observed in the

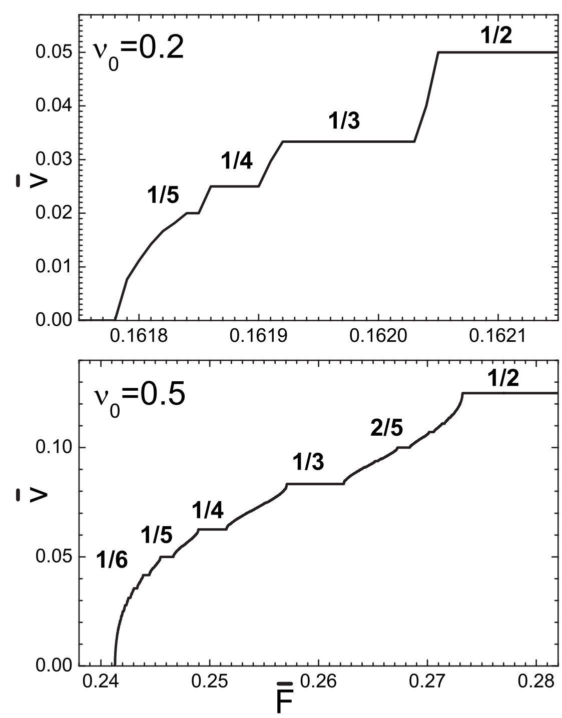

FIG. 9. Average velocity as a function of the average driving force for $\omega=\frac{1}{2}$, $K=4$, $F_{ac}=0.2$, $r=0.2$, and two different values of the frequency $\nu_0=0.2$ and 0.5.

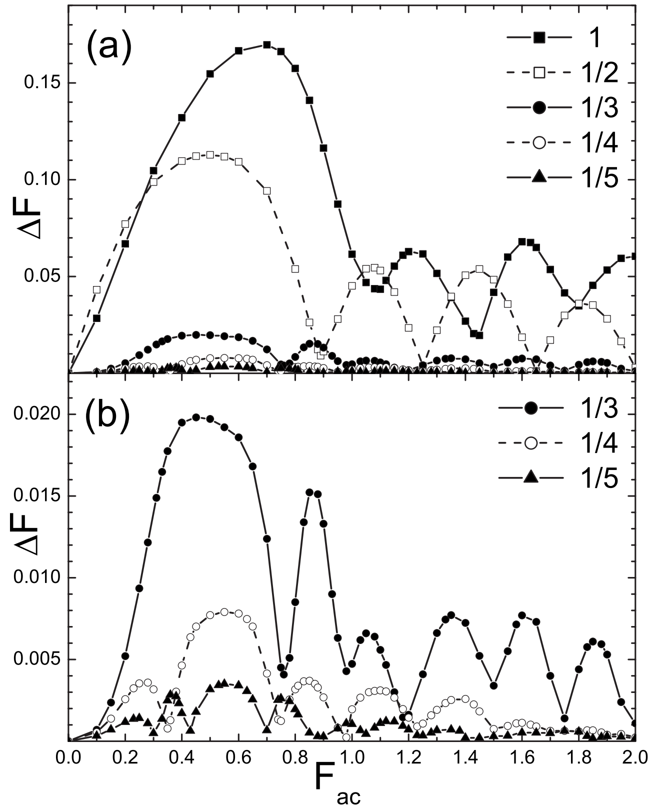

FIG. 10. (a) The width ΔF of the first harmonic, half-integer and subharmonic steps as a function of the ac amplitude for $\omega=\frac{1}{2}$, $K=4$, $\nu_0=0.5$, and $r=0.2$. The numbers $\frac{i}{m}$ mark the curves. (b) Enlarged curves for subharmonic steps $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.

two-dimensional Josephson-junction arrays where deviation from the Bessel-like behavior and reduction in second lobe is result of field-induced vortex super lattice and broken symmetry, and it cannot be obtained in single-junction case [25]. Since we have studied only one particular model, in order to get complete answer about behavior of Shapiro steps in realistic systems, other types of substrate potential have to be studied. These problems will be the subject of our future examinations.

IV. CONCLUSION

In this paper we have presented a detail study of the properties of Shapiro steps in the Frenkel-Kontorova model with deformable substrate potential. The obtained results have shown that deformation of the potential has strong influence on dynamical-mode-locking phenomena causing the appearance of large subharmonic steps and changing of stability and properties of harmonic steps. Harmonic and half-integer steps were found to be correlated in which, the larger width of half-integer steps, the smaller that of harmonic steps. In the amplitude dependence of harmonic steps, deformation and appearance of large half-integer steps will cause deviation from the Bessel like behavior to the oscillatory dependence where the second (even maxima) are lower than the third (odd) maxima. Strong influence of frequency on subharmonic steps has been observed where their number and size significantly increase at larger frequencies.

Presented results could be important for many areas of science such as studies of charge- or spin-density waves systems and Josephson-junction arrays that are motivated by fabrication of synchronization and superconducting devices. These models are closely related to the dissipative dynamics of the FK model [1,10]. Any application of interference phenomena and building of Shapiro step devices require a theoretical guideline for the observation of Shapiro steps. Our studies of interference effects in realistic models, and the analysis of the physical processes behind observed phenomena could bring an insight into the theory of Shapiro steps and contribute to the understanding of their behavior in real systems what is crucial for any technical application.

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